Homework

1. (1)Find the rank of the matrices.

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

(2)求下列矩阵的秩并判断它们是否有相同的相抵标准形.

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 0 & 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 & 5 \\ 0 & 3 & 3 \\ 0 & 2 & 2 \end{pmatrix}$$

(3)Find the rank of the matrix $\begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}$

where a, d, f are nonzero, and b, c, e are arbitrary numbers

2. If the rank of a 4×4 matrix A is 4 and the rank of a 5×3 matrix B is 3, what is the reduces row echelon form of A and B?($\mathrm{rref}(A) = ?$, $\mathrm{rref}(B) = ?$)

Homework

3. *let* $\vec{x} = \begin{bmatrix} 5 \\ 3 \\ -9 \end{bmatrix}$ *and* $\vec{y} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

(a) Find a *diagonal* matrix A such that $A\vec{x} = \vec{y}$.

(b) Find a matrix A of rank 1 such that $A\vec{x} = \vec{y}$.

(c) Find an *upper triangular* matrix A such that $A\vec{x} = \vec{y}$, where all the entries of A on and above the diagonal are nonzero.

(d) Find a matrix A with all nonzero entries such that $A\vec{x} = \vec{y}$.

4.(a) Consider a linear system of three equations with three unknowns. We are told that the system has a unique solution. What does the reduced row-echelon form of the coefficient matrix of this system look like? Explain your answer.

(b) Let A be a 4×4 matrix, and let $\vec{b}$ and $\vec{c}$ be two vectors in $\mathbb{R}^4$. We are told that the system $A\vec{x} = \vec{b}$ has a unique solution. What can you say about the number of solutions of the system $A\vec{x} = \vec{c}$?

(c) Let A be a 4×3 matrix, and let $\vec{b}$ and $\vec{c}$ be two vectors in $\mathbb{R}^4$. We are told that the system $A\vec{x} = \vec{b}$ has a unique solution. What can you say about the number of solutions of the system $A\vec{x} = \vec{c}$?

5.(a) Consider an $n \times m$ matrix A of rank n. Show that there exists an $m \times n$ matrix X such that $AX = I_n$. If $n < m$, how many such matrices X are there?

(b) Consider an $n \times n$ matrix A of rank n. How many $n \times n$ matrices X are there such that $AX = I_n$?

Homework(Optional)

1. A linear system of the form

$$A\vec{x} = \vec{0}$$

is called *homogeneous*. Justify the following facts:

a. All homogeneous systems are consistent.

b. A homogeneous system with fewer equations than unknowns has infinitely many solutions.

c. If $\vec{x}_1$ and $\vec{x}_2$ are solutions of the homogeneous system $A\vec{x} = \vec{0}$, then $\vec{x}_1 + \vec{x}_2$ is a solution as well.

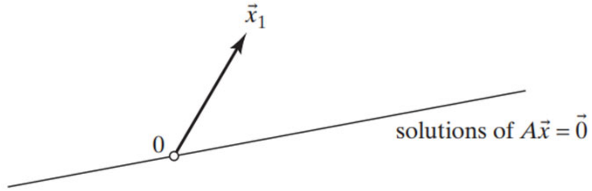
d. If $\vec{x}$ is a solution of the homogeneous system $A\vec{x} = \vec{0}$ and k is an arbitrary constant, then $k\vec{x}$ is a solution as well.

2. Consider a solution $\vec{x}_1$ of the linear system $A\vec{x} = \vec{b}$. Justify the facts stated in parts (a) and (b):

a. If $\vec{x}_h$ is a solution of the system $A\vec{x} = \vec{0}$, then $\vec{x}_1 + \vec{x}_h$ is a solution of the system $A\vec{x} = \vec{b}$.

b. If $\vec{x}_2$ is another solution of the system $A\vec{x} = \vec{b}$, then $\vec{x}_2 - \vec{x}_1$ is a solution of the system $A\vec{x} = \vec{0}$.

c. Now suppose A is a 2×2 matrix. A solution vector $\vec{x}_1$ of the system $A\vec{x} = \vec{b}$ is shown in the accompanying figure. We are told that the solutions of the system $A\vec{x} = \vec{0}$ form the line shown in the sketch. Draw the line consisting of all solutions of the system $A\vec{x} = \vec{b}$.



3. 方阵A被称为幂零矩阵，如果存在正整数k 使得$A^k = O$; 使 $A^k = O$成立的最小正整数k被称为A的幂零指数。证明或证否

（1）上(下)三角矩阵是幂零矩阵当且仅当它的主对角元全为0

（2）如果n级上(下)三角矩阵是幂零矩阵，那么它的幂零指数 $k \leq n$.

（3）若方阵A的任一正整数幂A^k的主对角线元素之和为0，则A是幂零矩阵.