Homework

1. Compute A^4. (1) $A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ (2) $A = \begin{pmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{pmatrix}$

2. Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$. Compute AB.

3. True or False

(1) Let A, B be two $n \times n$ matrices $(A+B)(A-B) = A^2 - B^2$

(2) If matrix $B_{k\times n}$ has columns, $\overrightarrow{b_1}, \overrightarrow{b_2}, \cdots, \overrightarrow{b_n}$, the matrix multiplication

$$A_{m\times k}B_{k\times n} = A(\overrightarrow{b_1} \quad \overrightarrow{b_2} \quad \cdots \quad \overrightarrow{b_n}) = (A\overrightarrow{b_1} \quad A\overrightarrow{b_2} \quad \cdots \quad A\overrightarrow{b_n})$$

(3) If the augmented matrices of two linear systems are column equivalent (transferred through elementary column operations), then the two systems have the same solution set.

Homework

4 Multiply these row exchange matrices in the order PQ, QP and P^2:

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } Q = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Find another non-diagonal matrix whose square $M^2 = I$.

5 Find the triangular matrix E, F that reduces Pascal's matrix to a smaller Pascal or to the identity matrix I:

$$E \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{pmatrix}, \qquad F \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

6 Write $M = \begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$ as a product of many factors $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Homework(optional)

1. Compute A^m

(1) $A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}_{n\times n}$

(2) $A = \begin{pmatrix} a & 1 & 0 & \cdots & 0 \\ 0 & a & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & a \end{pmatrix}_{n\times n}$

(3) $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Homework(optional)

2. Let $A = \begin{pmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & d_n \end{pmatrix}_{n\times n}$ where $d_1, d_2, \ldots, d_n$ are distinct.

Prove or disprove: If an $n \times n$ matrix B satisfying $AB = BA$, then B is diagonal.

3. Let A be an $n \times n$ matrix.

Prove or disprove: If $AB = BA$ for any $n \times n$ matrix B, then $A = cI$ where c is a constant.

4. 证明或证否:

若 A,B 是两个 n 阶斜对称矩阵(skew symmetric), 则 $AB-BA$也是斜对称矩阵.