Homework

1. If a matrix A is 5×3 and the product AB is 5×7, what is the size of B?

2. If possible, compute the matrix products.

(1) $\begin{pmatrix} 4 & 3 & 1 \\ 1 & -2 & 3 \\ 5 & 7 & 0 \end{pmatrix}\begin{pmatrix} 7 \\ 2 \\ 1 \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} -6 & 8 \\ 3 & -4 \end{pmatrix}$ (3) $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ (4) $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}\begin{pmatrix} -1 & 2 \end{pmatrix}$

(5) $\begin{pmatrix} 2 & 1 & 4 & 0 \\ 1 & -1 & 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 3 & 1 \\ 0 & -1 & 2 \\ 1 & -3 & 1 \\ 4 & 0 & -2 \end{pmatrix}$ (6) $\begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & 0 & -3 \end{pmatrix}$

(7) $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

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3. (1)Given $A = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$. Find A^{2005}. (2)Let $A = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$. Find A^n.

4. Let $A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}, B = \begin{pmatrix} 4 & -5 \\ 3 & k \end{pmatrix}$. If $AB = BA$, find all possible values of k.

5. Find all matrices that commute with the given matrix A.

(1) $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (2) $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

6. (1) Find all matrices X that satisfy $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(2) Find a matrix X that satisfy $X^{10} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Homework

提醒：$R^n = \{\vec{v} = (v_1, v_2, \cdots v_n) | v_i \in R\}$

Ex7. The *dot product* of two vectors

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \text{and} \quad \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

in $\mathbb{R}^n$ is defined by

$$\vec{x} \cdot \vec{y} = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n.$$

Note that the dot product of two vectors is a scalar. We say that the vectors $\vec{x}$ and $\vec{y}$ are *perpendicular* if $\vec{x} \cdot \vec{y} = 0$.

Find all vectors in $\mathbb{R}^3$ perpendicular to

$$\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}.$$

Draw a sketch.

Ex8. Find all vectors in $\mathbb{R}^4$ that are perpendicular to the three vectors

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 9 \\ 9 \\ 7 \end{bmatrix}.$$

Ex9. Find all solutions x_1, x_2, x_3 of the equation

$$\vec{b} = x_1 \vec{v}_1 + x_2 \vec{v}_2 + x_3 \vec{v}_3,$$

where

$$\vec{b} = \begin{bmatrix} -8 \\ -1 \\ 2 \\ 15 \end{bmatrix}, \vec{v}_1 = \begin{bmatrix} 1 \\ 4 \\ 7 \\ 5 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ 5 \\ 8 \\ 3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 4 \\ 6 \\ 9 \\ 1 \end{bmatrix}.$$