Homework

Ex1. Solve the linear systems using Gauss-Jordan elimination.

(1) $\begin{cases} 2x_1 + 4x_2 - 2x_3 = 2 \\ 4x_1 + 9x_2 - 3x_3 = 8 \\ -2x_1 - 3x_2 + 7x_3 = 10 \end{cases}$

(2) $\begin{cases} x_1 - 3x_2 + 4x_3 = -4 \\ 3x_1 - 7x_2 + 7x_3 = -8 \\ -4x_1 + 6x_2 - x_3 = 7 \end{cases}$

(3) $\begin{cases} 3x + 4y - z = 8 \\ 6x + 8y - 2z = 3 \end{cases}$

(4) $\begin{cases} x + y = 1 \\ 2x - y = 5 \\ 3x + 4y = 2 \end{cases}$

(5) $\begin{cases} -3y - 6z + 4w = 9 \\ -x - 2y - z + 3w = 1 \\ -2x - 3y + 3w = -1 \\ x + 4y + 5z - 9w = -7 \end{cases}$

(6) $\begin{cases} z + w = 0 \\ y + z = 0 \\ x + y = 0 \\ x + z = 0 \end{cases}$

Homework

Ex2 (1) Find an equation involving g, h and k that makes this augmented matrix correspond to a consistent system.

$$\begin{pmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{pmatrix}$$

(2) Determine the value(s) of a such that the linear system is consistent.

$$\begin{cases} 3x_1 + x_2 - x_3 - 2x_4 = 2 \\ x_1 - 5x_2 + 2x_3 + x_4 = -1 \\ 2x_1 + 6x_2 - 3x_3 - 3x_4 = a + 1 \\ -x_1 - 11x_2 + 5x_3 + 4x_4 = -4 \end{cases}$$

Homework

Ex3 We say that two $n \times m$ matrices in reduced row-echelon form are of the same type if they contain the same number of leading 1's in the same positions. For example,

$$\begin{bmatrix} \textcircled{1} & 2 & 0 \\ 0 & 0 & \textcircled{1} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \textcircled{1} & 3 & 0 \\ 0 & 0 & \textcircled{1} \end{bmatrix}$$

are of the same type. How many types of 2×2 matrices in reduced row-echelon form are there?

How many types of 3×2 matrices in reduced row-echelon form are there?

How many types of 2×3 matrices in reduced row-echelon form are there?

Homework

Ex4 Suppose matrix A is transformed into matrix B by means of an elementary row operation. Is there an elementary row operation that transforms B into A? Explain.

Suppose matrix A is transformed into matrix B by a sequence of elementary row operations. Is there a sequence of elementary row operations that transforms B into A? Explain your answer.

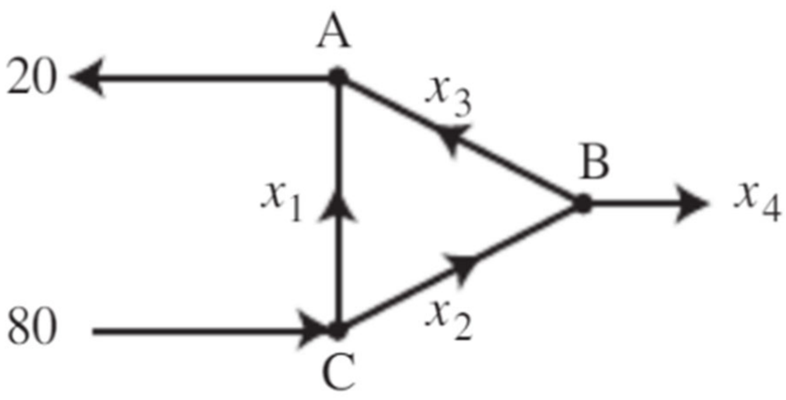
Homework

Ex5 Find a polynomial of degree ≤ 2 [of the form $f(t) = a + bt + ct^2$] whose graph goes through the points $(1, p)$, $(2, q)$, $(3, r)$, where p, q, r are arbitrary constants. Does such a polynomial exist for all values of p, q, r?

Ex6 Some parking meters in Milan, Italy, accept coins in the denominations of 20¢, 50¢, and €2. As an incentive program, the city administrators offer a big reward (a brand new Ferrari Testarossa) to any meter maid who brings back exactly 1,000 coins worth exactly €1,000 from the daily rounds. What are the odds of this reward being claimed anytime soon?

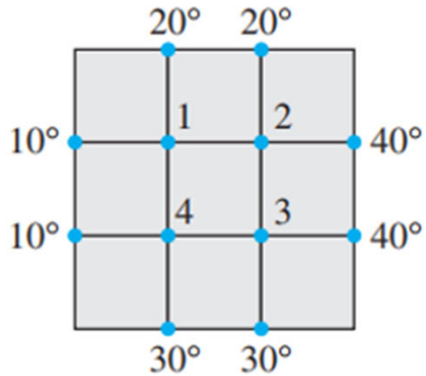
Homework

Ex7.Find the general flow pattern of the network shown in the figure. Assuming that the flows are all nonnegative, what's the largest possible value for x_3.



Ex8. An important concern in the study of heat transfer is to determine the steady-state temperature distribution of a thin plate when the temperature around the boundary is known. Assume the plate shown in the figure represents a cross section of a metal beam, with negligible heat flow in the direction perpendicular to the plate. Let $T_1, \dots, T_4$ denote the temperatures at the four interior nodes of the mesh in the figure. The temperature at a node is approximately equal to the average of the four nearest nodes—to the left, above, to the right, and below.[2] For instance,

$$T_1 = (10 + 20 + T_2 + T_4)/4, \quad \text{or} \quad 4T_1 - T_2 - T_4 = 30$$



a. Write a system of four equations whose solution gives estimates for the temperatures $T_1, \dots, T_4$.

b. Solve the system of equations from Exercise a.. [*Hint:* To speed up the calculations, interchange rows 1 and 4 before starting "replace" operations.]

Homework

Ex9. Solve the system in n variables whose augmented matrix is

$$A = \begin{pmatrix} 0 & 1 & 1 & \cdots & 1 & 0 \\ 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 0 & 1 \end{pmatrix}_{n\times(n+1)}$$

Challenge Problems (optional)

Ex10. Solve the linear system in n variables whose augmented matrix is

$$(1)\quad A=\begin{pmatrix} 1+a_1 & 1 & 1 & \cdots & 1 & b \\ 1 & 1+a_2 & 1 & \cdots & 1 & b \\ 1 & 1 & 1+a_3 & \cdots & 1 & b \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 1+a_n & b \end{pmatrix}_{n\times(n+1)}$$

where $a_i \neq 0, i=1,2,\dots,n$ and $\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}\neq -1$.

$$(2)\quad A=\begin{pmatrix} 1 & 2 & 3 & \cdots & n & b \\ n & 1 & 2 & \cdots & n-1 & b \\ n-1 & n & 1 & \cdots & n-2 & b \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 2 & 3 & 4 & \cdots & 1 & b \end{pmatrix}_{n\times(n+1)}$$